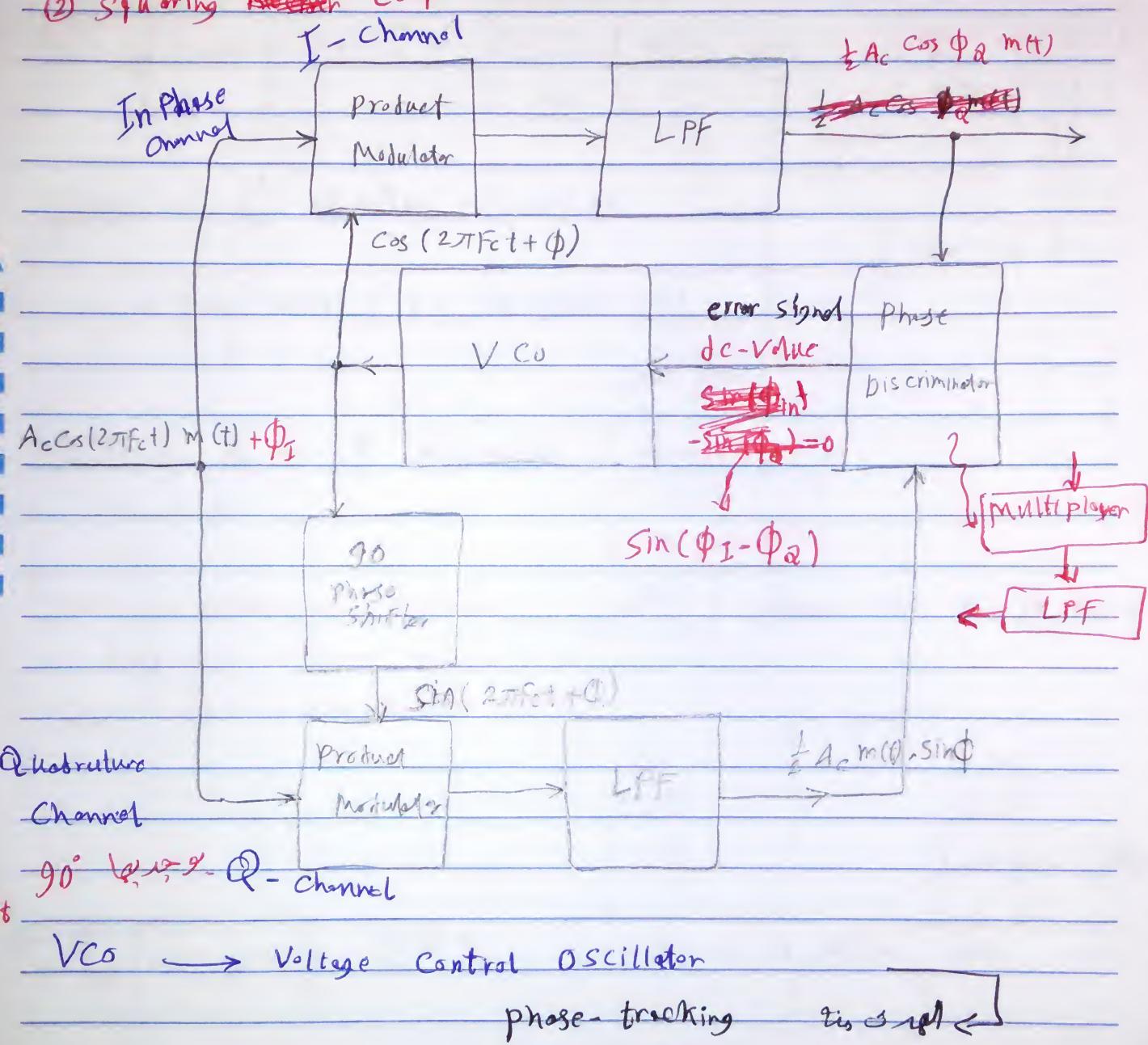


## Using a phase-shift feedback

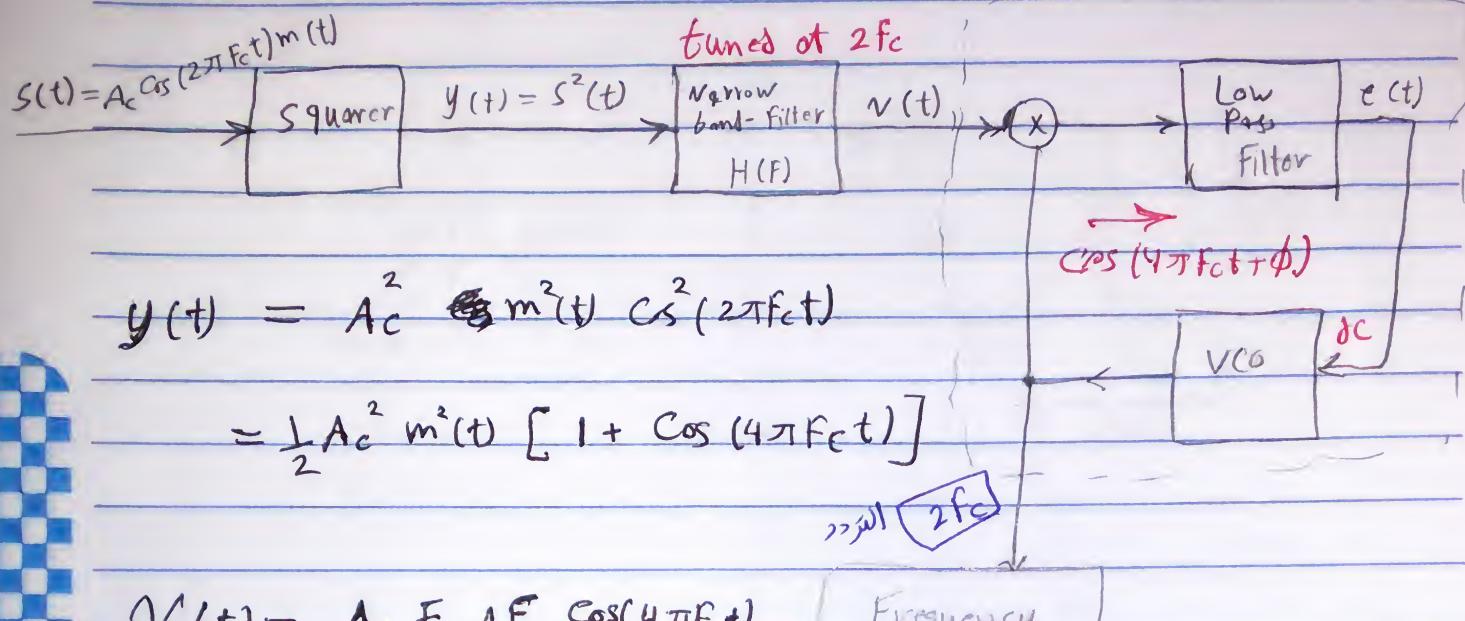
① Costas Receiver

② Synchronizing Loop



## ② Squaring Loop

PLL



$$y(t) = A_c^2 m^2(t) \cos^2(2\pi f_c t)$$

$$= \frac{1}{2} A_c^2 m^2(t) [1 + \cos(4\pi f_c t)]$$

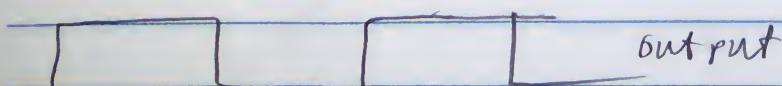
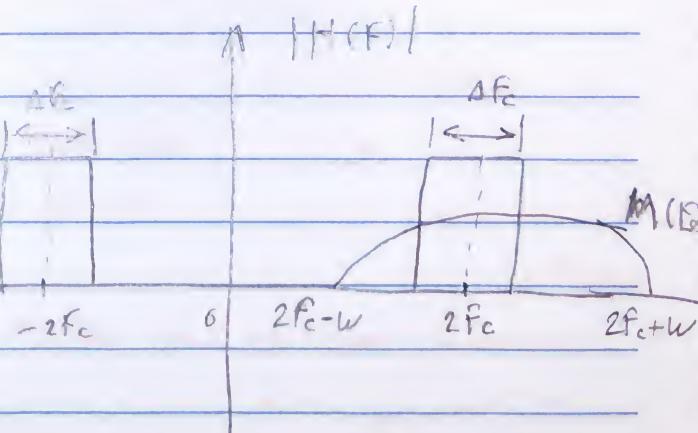
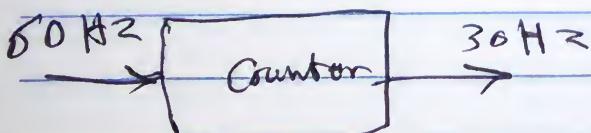
$e(t) \rightarrow$  difference between

frequency (phase  $n(t)$ )

and frequency and phase of VCO

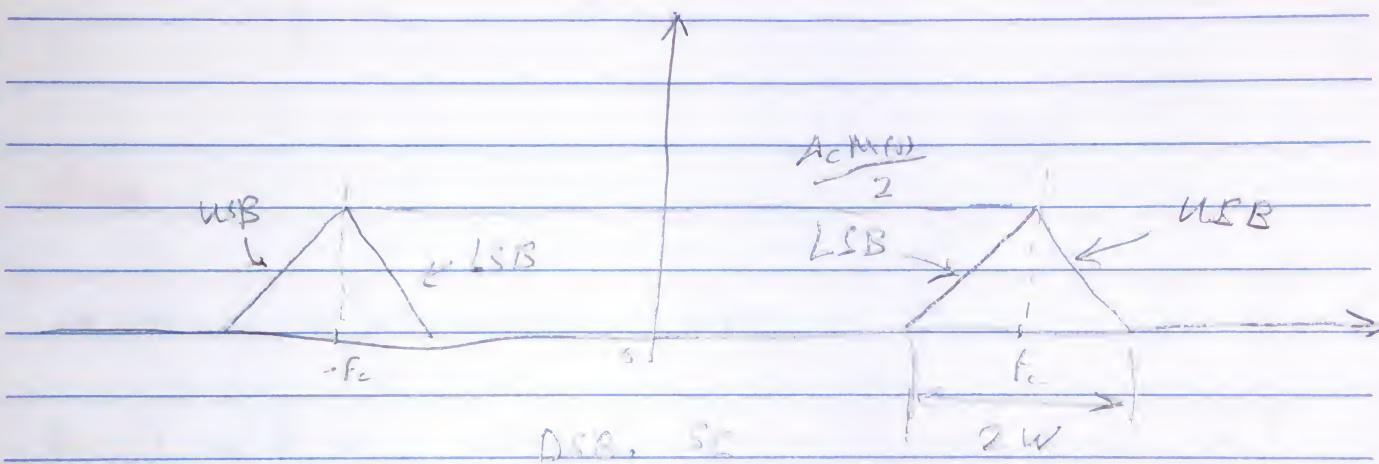
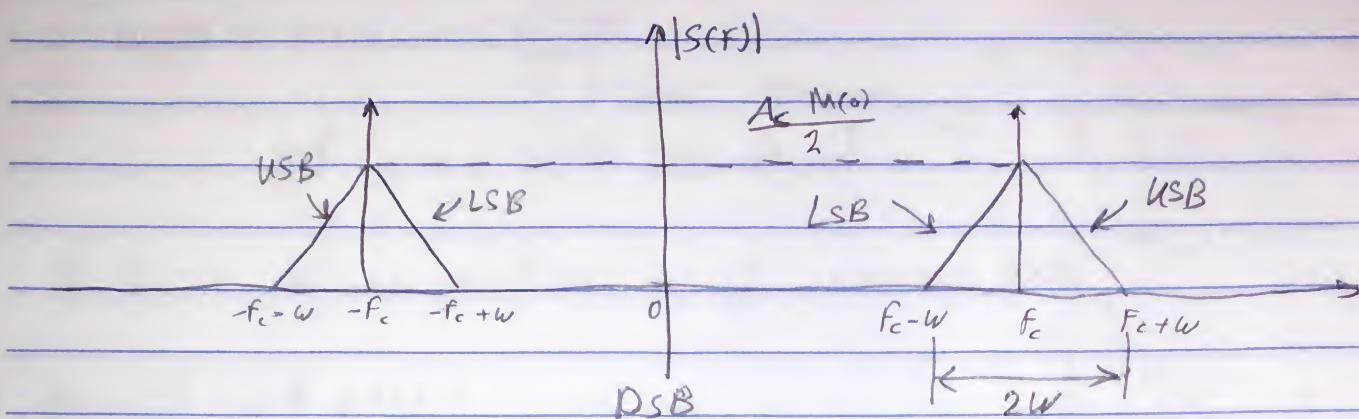
Carrier wave at  $f_c$  frequency

Frequency  
divider  
by 2  
Counter



6 bit  $\otimes 0$

# Single Side band Modulation (SSB)



Single sideband  $\Rightarrow$

Band width  $\Rightarrow$   $\frac{1}{2} W$  \*

Power  $\Rightarrow$   $\frac{1}{2} P_{avg}$  \*

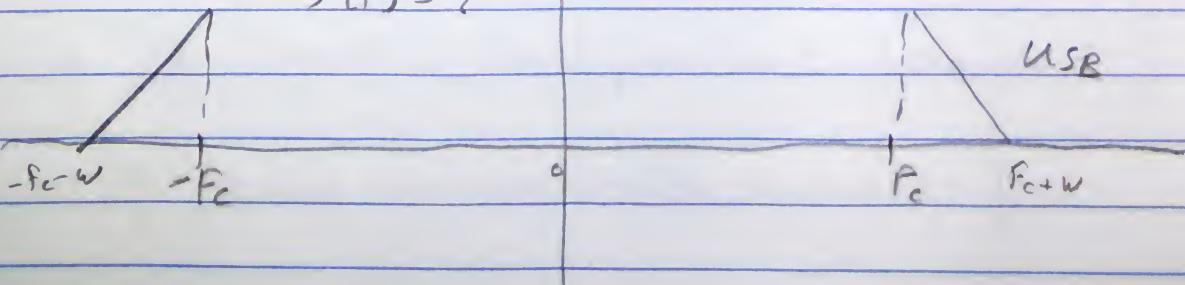
$\Rightarrow \frac{1}{2} P_{avg}$

Perfect band pass filter  $\Rightarrow$   $\frac{1}{2} W$  \*

$\Rightarrow$   $\frac{1}{2} P_{avg}$  \*

$$S(t) = ?$$

$$S(f) = ?$$



pre-envelope of  $m(t)$ :

$$m_+(t) = m(t) + j \hat{m}(t)$$

↓ Hilbert transform

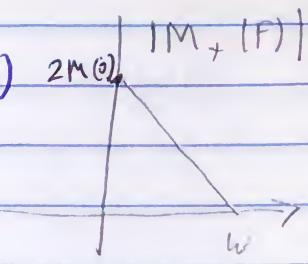
$90^\circ$  phase shift  $\leftrightarrow \leftrightarrow$

$$m(0) M(f)$$



$$\hat{m}(t) = m(t) \text{ shifted by } \frac{\pi}{2} \text{ (Phase)} \quad 2M(0)$$

$$M_+(f) = \begin{cases} 2M(f) & f > 0 \\ 0 & f < 0 \end{cases}$$

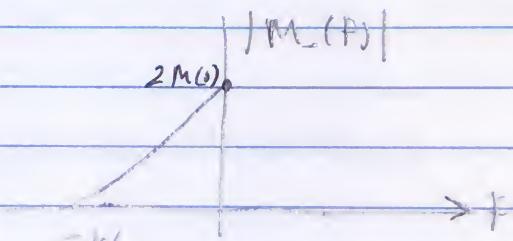


~~Redacted~~

$$m_-(t) = m_+^*(t) = m(t) - j \hat{m}(t)$$

$$\hat{M}(f) = M(f) \cdot e^{j \frac{\pi}{2}}$$

$$M_-(f) = \begin{cases} 0 & f > 0 \\ 2M(f) & f < 0 \end{cases}$$



To get single side band:

up or side band

$$s(t) = [m_+(t) e^{j2\pi f_c t} + m_-(t) e^{-j2\pi f_c t}] \frac{A_c}{4}$$

$$s(t) = \frac{A_c}{4} [(m(t) + j \hat{m}(t)) (\cos(2\pi f_c t) + j \sin(2\pi f_c t))]$$

$$+ \frac{A_c}{4} [(m(t) - j \hat{m}(t)) (\cos(2\pi f_c t) - j \sin(2\pi f_c t))]$$

$$= \frac{A_c}{4} [2m(t) \cos(2\pi f_c t) - 2\hat{m}(t) \sin(2\pi f_c t)]$$

$$= \frac{A_c}{2} [m(t) \cos(2\pi f_c t) - \hat{m}(t) \sin(2\pi f_c t)]$$



Lower Side Band

$$S(t) = \frac{A_c}{2} \left[ m(t) \cos(2\pi f_c t) + \hat{m}(t) \sin(2\pi f_c t) \right]$$

For single side band

$$S(t) = \frac{A_c}{2} \left[ m(t) \cos(2\pi f_c t) + \hat{m}(t) \sin(2\pi f_c t) \right]$$

Ex:  $m(t) = A_m \cos(2\pi f_m t)$

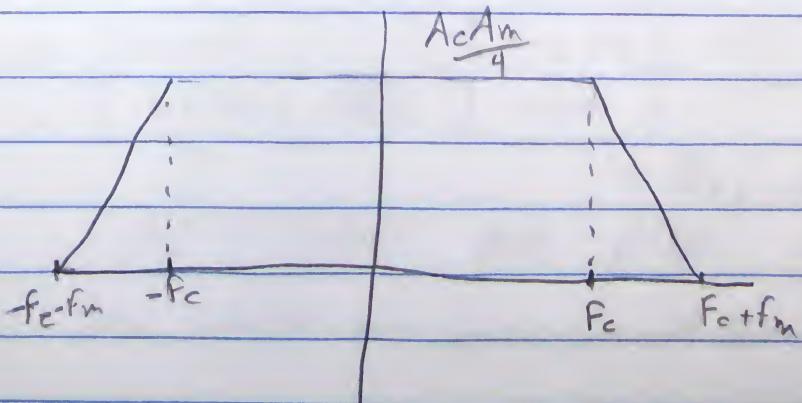
Ans USB Am ~~Am~~

$$\hat{m}(t) = A_m \cos(2\pi f_m t + \frac{\pi}{2}) = A_m \sin(2\pi f_m t)$$

$$S(t) = \frac{A_c}{2} \left[ A_m \cos(2\pi f_m t) \cos(2\pi f_c t) - A_m \sin(2\pi f_m t) \sin(2\pi f_c t) \right]$$

$$= \frac{A_c A_m}{2} \cos(2\pi (f_m + f_c)t)$$

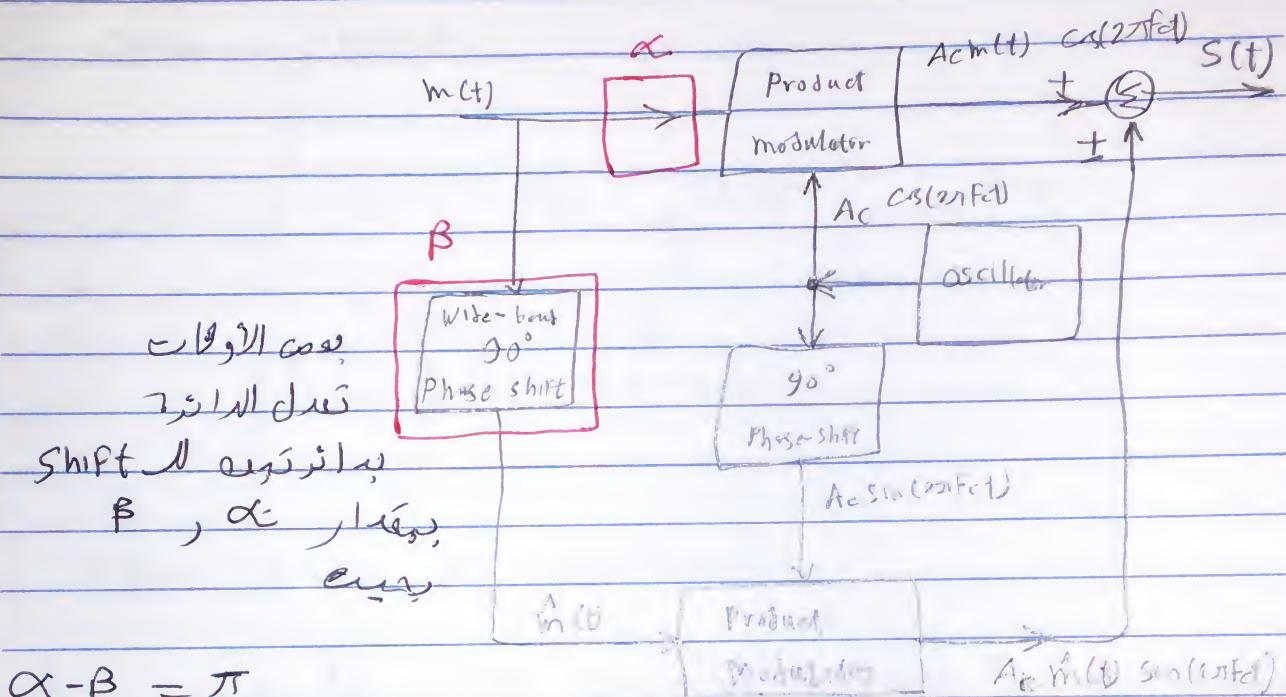
$$S(f) = \frac{A_c A_m}{4} \left[ \delta(f + f_c + f_m) + \delta(f - f_c - f_m) \right]$$



### Generation of SSB Waves :

① phase discriminator

② Frequency discriminator



$$\alpha - \beta = \frac{\pi}{2}$$

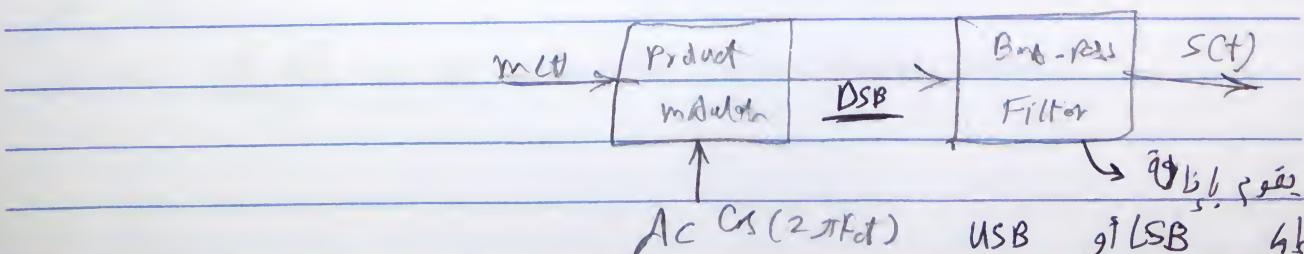
مودولیشن کے لیے

$m(t) \propto \cos(\omega_m t)$

$m(t)$  کا ایک جو ایک دوسرے کے مقابلے میں

Phase discriminator

Frequency discriminator

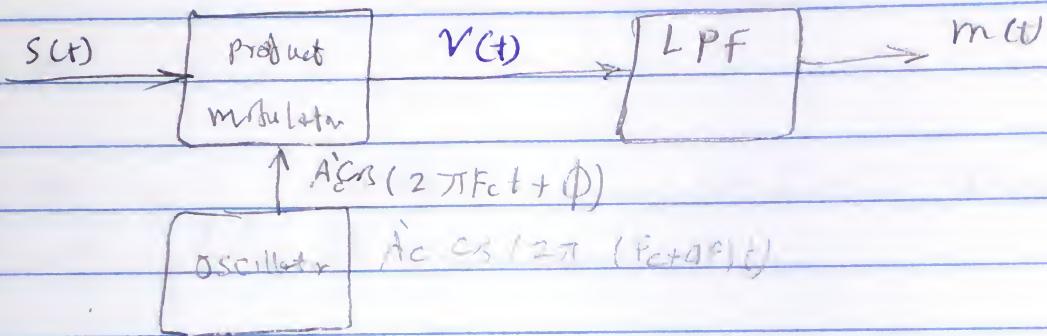


لیکن یہی ایک بہت سیل ایکسپلائیٹر

De modulation of SSB :

$$S(t) = \frac{A_c}{2} [ m(t) \cdot \cos(2\pi f_c t) \mp \overset{1}{m}(t) \sin(2\pi f_c t) ]$$

Coherent Detector



$$V(t) = S(t) \cdot A_c \cos(2\pi f_c t)$$

$$= \frac{A_c A_c'}{2} [ m(t) \cos(2\pi f_c t) \cos(2\pi f_c t) \mp \overset{1}{m}(t) \sin(2\pi f_c t) \cos(2\pi f_c t) ]$$

$\cos^2(2\pi f_c t)$

$\overset{1}{m}(t)$   
 $\sin(4\pi f_c t)$

$$= \frac{A_c A_c'}{2} \left[ \frac{1}{2} m(t) [1 - \cos(4\pi f_c t)] \right]$$

$$= \frac{A_c A_c'}{4} m(t) \cos \phi \rightarrow \text{Phase error}$$

## Angle Modulation

$$S(t) = A_c \cos(\theta_i(t))$$

Am wave

$$S(t) = A_c [1 + k_p m(t)] \cos(2\pi f_c t + \phi)$$

$\theta_i(t)$

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} = \text{instantaneous frequency}$$

FM wave  $\rightarrow F$

PM wave  $\rightarrow \phi$

$$S(t) = A_c \cos(2\pi f_c t + \phi_c)$$

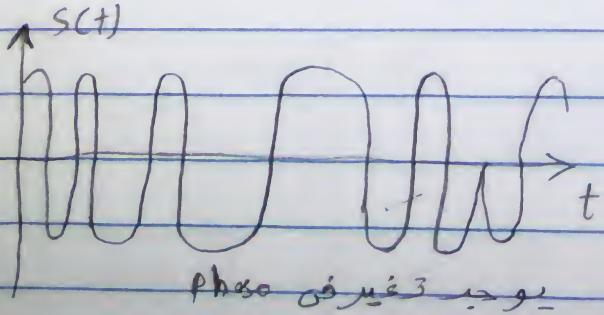
~~Frequency~~  $\rightarrow$  Un modulated Frequency  $\rightarrow$  Un modulated Angle

$$\theta_i(t) = 2\pi f_c t + \phi_c$$

$$\text{PM wave} \rightarrow \theta_i(t) = 2\pi f_c t + k_p m(t)$$

$k_p \rightarrow$  Phase Sensitivity

$$S(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$



$$f_i(t) = \frac{1}{2\pi} \frac{d\phi_i(t)}{dt}$$

$$\phi_i(t) = 2\pi \int_0^t f_i(t) dt$$

$$FM \rightarrow f_i(t) = f_c + k_p \cdot m(t)$$

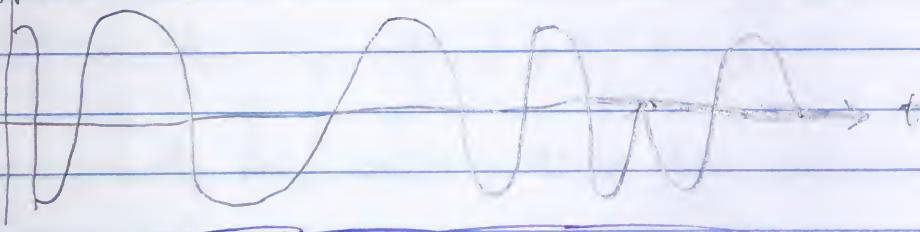
$k_p$  → frequency sensitivity

$$\phi_i(t) = 2\pi \int_0^t [f_c + k_p m(t)] dt$$

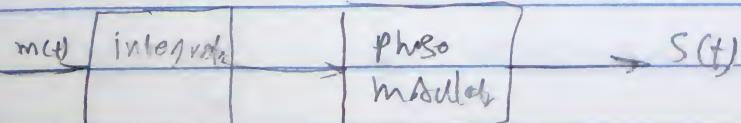
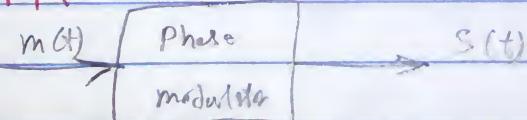
$$\phi_i(t) = 2\pi f_c t + 2\pi k_p \int_0^t m(t) dt$$

$$s(t) = A_c \cos [2\pi f_c t + 2\pi k_p \int_0^t m(t) dt]$$

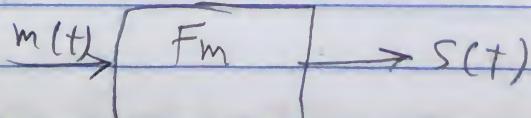
$\uparrow$  SCA



FM from PM



PM Pm FM



FM and PM  $\rightarrow$  better than  $\rightarrow$  AM  
 noise  $\rightarrow$  min, max  $\rightarrow$  less noise  
 high Amplitude  $\rightarrow$  V

Widely used, widely used

FM Type:

- ① Single tone FM
- ② Multi tone FM

\* Single tone:

$$m(t) = A_m \cos(2\pi f_m t)$$

$$s(t) = A_c \cos \left[ 2\pi f_c t + 2\pi k_p \int_0^t m(t) dt \right]$$

$$= A_c \cos \left[ 2\pi f_c t + 2\pi k_p \frac{A_m}{2\pi f_m} \sin(2\pi f_m t) \right]$$

$\Delta f = \pm k_p A_m$  ~~Frequency~~ ~~deviation~~

$$s(t) = A_c \cos \left( 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t) \right)$$

$$\beta = \frac{\Delta f}{f_m} \rightarrow \text{modulation index}$$

$$\text{Frange} = f_c \pm k_p A_m = f_c \pm \Delta f$$

$$s(t) = A_c \cos \left[ 2\pi f_c t + \beta \sin(2\pi f_m t) \right]$$

$$\beta \rightarrow \begin{cases} \text{Narrow Band FM} & \beta < 1 \\ \text{Wide Band FM} & \beta > 1 \end{cases}$$

Narrow Band FM wave

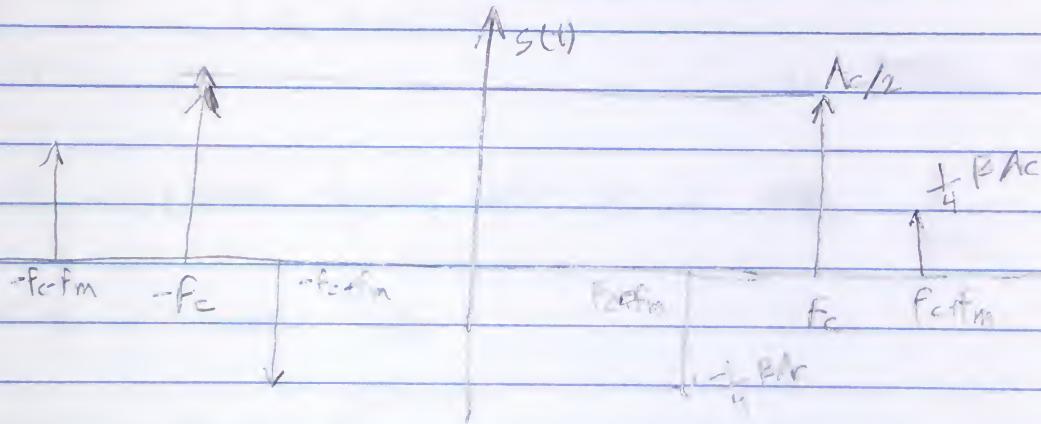
$\beta$  is very small

$$s(t) = A_c \cos(2\pi f_c t) \cos[\beta \sin(2\pi f_m t)]$$

$$- A_c \sin(2\pi f_c t) \sin[\beta \sin(2\pi f_m t)]$$

$$\simeq \beta \sin(2\pi f_m t)$$

$$s(t) = A_c \cos(2\pi f_c t) - A_c \beta \sin(2\pi f_c t) \sin(2\pi f_m t)$$



$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} \beta A_c [\cos(2\pi(f_c + f_m)t) - \cos(2\pi(f_c - f_m)t)]$$

$$B.W = 2f_m$$

## \* Wide Band frequency Modulation

$$\beta \gg 1$$

$$s(t) = A_c \cos [2\pi f_c t + \beta \sin (2\pi f_m t)]$$

$$= A_c \operatorname{Re} \{ e^{j2\pi f_c t + \beta \sin (2\pi f_m t)} \}$$

$$= A_c \operatorname{Re} \{ e^{j2\pi f_c t} \cdot e^{j\beta \sin (2\pi f_m t)} \}$$

$$\hat{s}(t) = e^{j\beta \sin (2\pi f_m t)} \rightarrow \text{Complex Envelope}$$

$\hat{s}(t) \rightarrow$  periodic signal with period  ~~$\frac{1}{2f_m}$~~   $\frac{1}{f_m}$

Using Complex F.S.

$$\hat{s}(t) = \sum C_n e^{j2\pi n f_m t}$$

$$C_n = \frac{1}{2f_m} \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \hat{s}(t) \cdot e^{-j2\pi n f_m t} dt$$

$$= f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} e^{j\beta \sin (2\pi f_m t)} \cdot e^{-j2\pi n f_m t} dt$$

$$= f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} e^{j[\beta \sin (2\pi f_m t) - 2\pi n f_m t]} dt$$

$$x = 2\pi f_m t$$

$$dx = 2\pi f_m dt$$

$$c_n = f_m \int_{-\pi}^{\pi} e^{j[\beta \sin(x) - nx]} dx$$

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j[\beta \sin(x) - nx]} dx \quad \text{Bessel Function}$$

$$c_n = J_n(\beta)$$

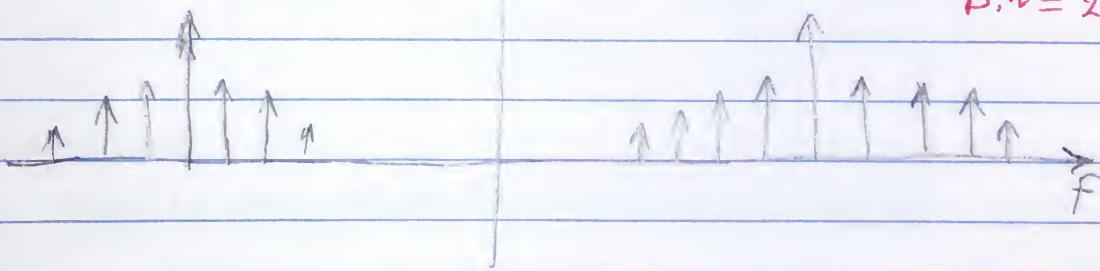
$$S(t) = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi f_m t}$$

$$S(t) = A_c \operatorname{Re} \left\{ e^{j2\pi f_m t} \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi f_m t} \right\}$$

$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_m + f_m)t)$$

$|S(p)|$

$$B.W = 2\Delta f + 2f_m$$



$J_n(\beta)$  properties

$$n=0$$

$$J_0(\beta) = 1$$

$$n = \text{even}$$

$$J_n(\beta) = J_{-n}(\beta)$$

$$n = \text{odd}$$

$$J_n(\beta) = -J_{-n}(\beta)$$

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

$$J_n(\beta) = (-1)^n J_{-n}(\beta)$$

## \* Generation of FM waves:

Transmission band width of FM wave:

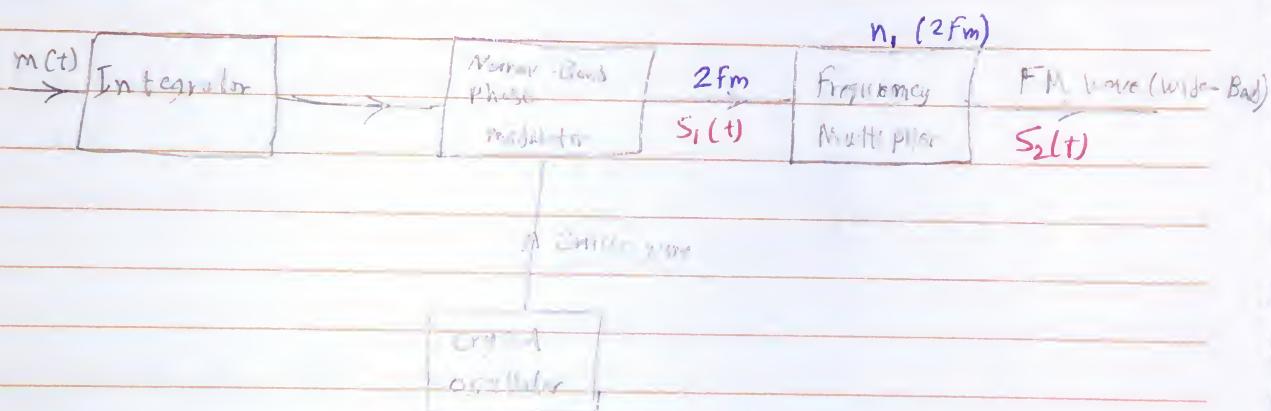
Corson's rule (Wide Band FM)

$$B.W. \approx 2\Delta F + 2f_m \approx 2n_{max}f_m$$

$$|J_n(B)| > 0.01$$

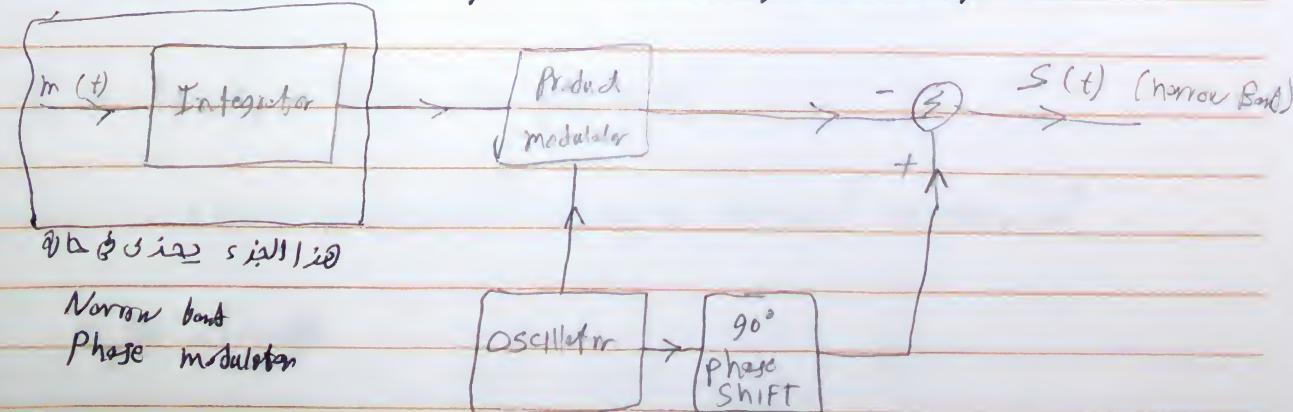
$$\Delta F = \frac{K_f A_m}{f_m}$$

### \* Indirect ~~mod~~ FM generator:



### Narrow Band (FM)

$$S(t) = A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$$

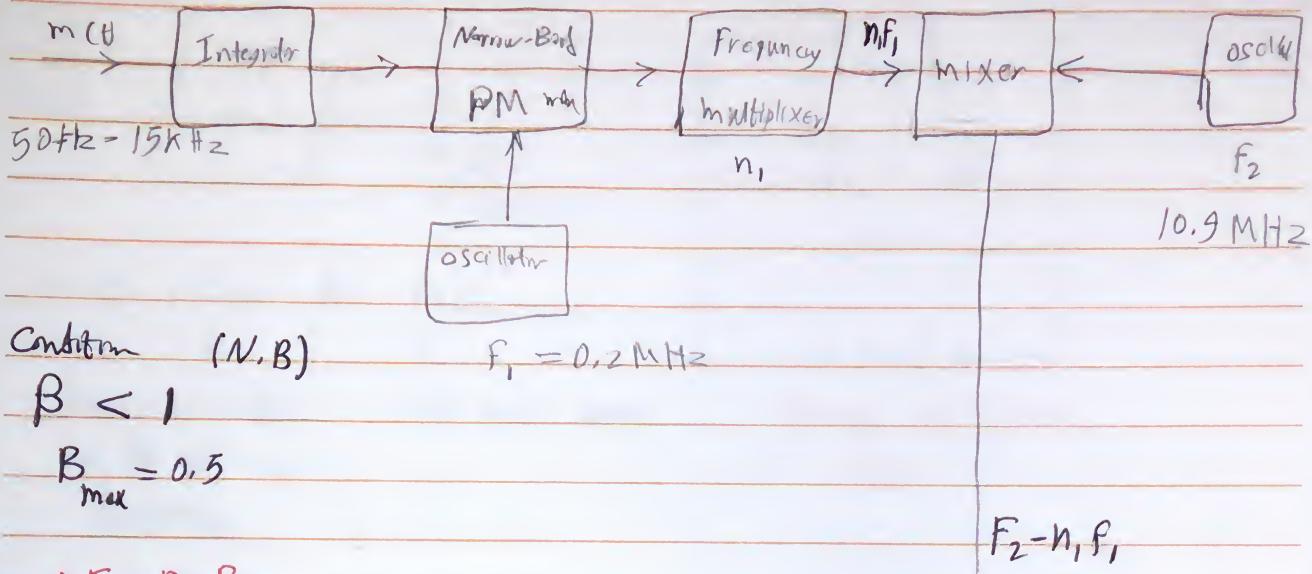


$$S_1(t) = A_1 \cos[2\pi f_c t + 2\pi K_f \int m(t) dt] \quad \text{if } m(t) = A_m \cos(2\pi f_m t)$$

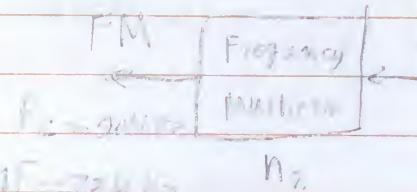
$$S_1(t) = A_1 \cos[2\pi f_c t + \beta_1 \sin(2\pi f_m t)] \quad \beta_1 \ll 1$$

$$S_2(t) = A_1 \cos[2\pi f_c t + n_1 \beta_1 \sin(2\pi f_m t)] \quad n_1, \beta_1 \gg 1$$

Example : Find  $n_1, n_2$



$$\Delta F_1 = 0.5 * 50 \text{ Hz} = 25 \text{ Hz}$$



$$\Delta F_{\text{prod}} = n_1 \cdot n_2 \cdot \Delta F_1 = 75 \text{ kHz}$$

$$\boxed{n_1 \cdot n_2 = 3000}$$

$$F_C = -n_1 n_2 F_1 + n_2 F_2 = 90 * 10^6$$

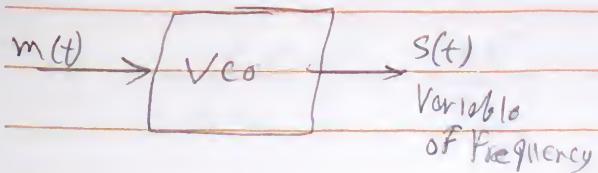
$$-3000 * 0.2 * 10^6 + n_2 * 10.9 * 10^6 = 90 * 10^6$$

$$n_2 \approx 63$$

$$n_1 \approx 48$$

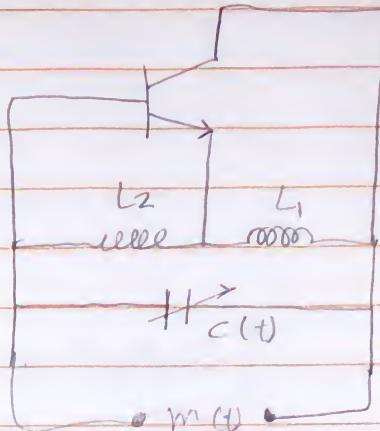
\* Direct methods Direct FM generator

\* Voltage Controlled oscillator



$$C(t) = C_0 + \Delta C m(t)$$

Capacitance in  
the absence of  
modulation



base band signal

Hartley oscillator

VCO

$\Delta C \rightarrow$  max. Change of Capacitance

It's a small pic

P.N Junction with reverse bias

  $\rightarrow$  Varactor or Varicap

$$f_i = \frac{1}{2\pi \sqrt{(L_1 + L_2) C(t)}} = \frac{1}{2\pi \sqrt{(L_1 + L_2) [C_0 + \Delta C m(t)]}}$$

$$1P m(t) = \cos(2\pi f_m t)$$

~~$$f_i = \frac{1}{2\pi \sqrt{(L_1 + L_2) C_0}}$$~~

$$f_i(t) = f_0 \left[ 1 + \frac{\Delta C}{C_0} \cos(2\pi f_m t) \right]^{-\frac{1}{2}}$$

$$f_0 = \frac{1}{2\pi \sqrt{(L_1 + L_2) C_0}}$$

$$F_i(t) = F_0 \left[ 1 + \frac{\Delta C}{C_0} \cos(2\pi f_m t) + \dots \right]$$

$$\boxed{\Delta C < C_0}$$

$$\frac{-\Delta C}{2C_0} = \frac{\Delta F}{f_m}$$

$$\therefore F_i(t) = F_0 \left[ 1 + \frac{\Delta F}{f_m} \cos(2\pi f_m t) \right]$$

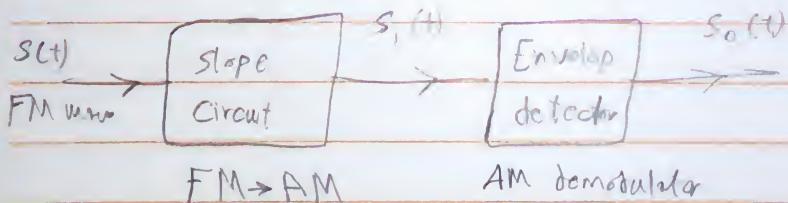
Instantaneous Frequency of FM wave

Disadvantage  $\rightarrow$  \* Unstable Frequency  
\* Variable Capacitor is not Linear

\* Detection of FM waves

- ① Slope detector
- ② PLL  $\rightarrow$  Phase Locked - Loop

① Slope detector



Slope Circuit  $\rightarrow$  Differentiator

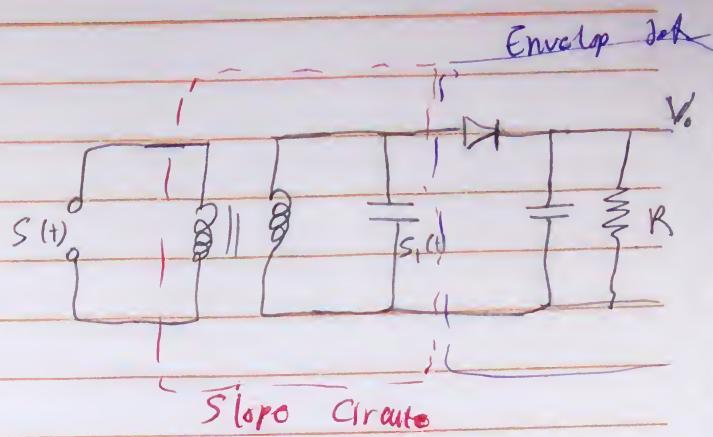
$$s(t) = A_c C_0 \left[ 2\pi f_c t + 2\pi K_f \int_0^t m(t) dt \right]$$

$$s_i(t) = -A_c \left[ 2\pi f_c t + 2\pi K_f \int_0^t m(t) dt \right] \sin \left[ 2\pi f_c t + 2\pi K_f \int_0^t m(t) dt \right]$$

$$s_o(t) = -A_c \left[ 2\pi f_c t + 2\pi K_f \int_0^t m(t) dt \right]$$

$$s_o(t) \propto m(t)$$

Slope Circuite



② phase - Locked - Loop :

